PhDs in Logic VII

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Contents

Program

Thursday, May 14

8:30	Opening		
8:45–9:30	Thomas Eiter	Querying Ontology Knowledge Bases using Datalog	(p. 9)
9:30–10:00	Break		
10:00–10:30	Sylwia Polberg	Intertranslatability of Abstract Argumentation Frameworks	(p. 22)
10:30–11:00	Jesse Heyninck	Combining Different Forms of Defeasible Reasoning in Abstract Argumentation: Integrating Mechanisms from Adaptive Logics	(p. 26)
11:00–11:30	Zhiguang Zhao	An Abstract Algebraic Logic view of propositional-attitude aggregation theory	(p. 30)
11:30–12:15	Thomas Eiter	Towards a Logic-Based Framework for Analyzing Stream Reasoning	(p. 10)
12:15–13:30	Lunch break		
13:30–14:30	Torsten Schaub	Answer Set Solving in a nutshell	(p. 11)
14:30–15:00	Matteo Pascucci	Basic model theory of modal languages with propositional constants	(p. 33)
15:00–15:30	Przemysław Andrzej Wałęga	Modal Logics for Qualitative Spatial Reasoning	(p. 36)
15:30–16:00	Julia Ilin	Algebra-based canonical formulas for intermediate logics	(p. 40)
16:00–16:30	Break		
16:30–17:15	Uli Sattler	Complexity in Logic: off the beaten track towards pastures new (1)	(p. 12)
17:15–17:45	Wolfgang Fischl and Shqiponja Ahmetaj	Towards Reconciling SPARQL and Certain Answers	(p. 43)
17:45–18:15	Gergely Buday	Verifying the SECD machine in Nominal Isabelle	(p. 46)
20:15		Unofficial event: Vienna Philharmonic Summer Night Concert	

Friday, May 15

9:00–9:45	Uli Sattler	Complexity in Logic: off the beaten track towards pastures new (2)	(p. 12)
9:45–10:15	Break		
10:15–10:45	Michael Kompatscher	Dichotomy results for constraint satisfaction problems	(p. 51)
10:45–11:15	Eduard Eiben	MSO model checking of graphs with unbounded rank-width	(p. 53)
11:15–11:30	VCLA International Student Awards Ceremony		
11:30–12:00	VCLA International Student Awards: Sophie Spirkl		(p. 17)
12:00–12:30	VCLA International Student Awards: Luke Schaeffer		(p. 18)
12:30–14:00	Lunch break		
14:00–14:45	Michael Moortgat	New directions in typelogical semantics (1)	(p. 13)
14:45–15:15	VCLA International Student Awards: Kuldeep S. Meel		(p. 19)
15:15–15:45	VCLA International Student Awards: Pablo Munoz		(p. 20)
15:45–16:15	Break		
16:15–17:00	Michael Moortgat	New directions in typelogical semantics (2)	(p. 13)
17:00–17:30	Diana Carolina Montoya Amaya	Some Cardinal Invariants of the Generalized Baire Spaces	(p. 56)
17:30–18:00	Marlene Koelbing	Gaps in ω^ω	(p. 58)
18:00–18:30	Anda-Ramona Tanasie	The lifting problem for measure and category	(p. 60)
20:00		Conference Dinner at Heuriger Werner-Welser (http://www.werner-welser.at/)	

Saturday, May 16

9:45–10:45	Revantha Ramanayake	Cut-elimination in generalisations of the sequent calculus	(p. 14)
10:45–11:15	Break		
11:15–11:45	Apostolos Tzimoulis	Unified correspondence as a proof-theoretic tool	(p. 62)
11:45–12:15	Daniel Günzel	Logical Metatheorems for Abstract Spaces axiomatized in Positive Bounded Logic	(p. 66)
12:15–12:45	Daniel Koernlein	Proof Mining in Nonlinear Analysis	(p. 70)
12:45–13:45	Lunch break		
13:45–14:15	Christof Spanring	On the Why and How of implicit conflicts in Abstract Argumentation	(p. 72)
14:15–14:45	Esther Anna Corsi	Extracting consequence relations from abstract argumentation frames	(p. 76)
14:45–15:15	Raine Rönnholm	The Expressive Power of k-ary Inclusion-Exclusion Logic	(p. 79)
15:15	Closing		

Invited Talks

Querying Ontology Knowledge Bases using Datalog

Thomas Eiter
Knowledge Based Systems Group, TU Wien

Description logics play a dominant role in the formalization of ontologies for practical applications of computer science, and they provide, for instance, the formal underpinning of the Web Ontology Language (OWL) recommended by the World Wide Web consortium (W3C). While description logics had been initially targeted at reasoning about conceptual knowledge, in the last decade querying description logic knowledge bases (often simply called ontologies) to elicit information about individuals similar as from databases has received growing attention. Different techniques for query answering have been proposed, among them also to query rewriting to Datalog, which is a well-established query language for relational databases that has several effective reasoning engines available. We consider particular query types, instance and conjunctive queries, which for several description logics have been transformed to Datalog, among them ones underlying the OWL 2 Profiles. These transformations can also be beneficially exploited for extensions of ontologies, such as in combinations with rules, and have led to research prototype implementations. To exploit and extend the Datalog approach to query answering is an active area, which provides a number of research opportunities.

Towards a Logic-Based Framework for Analyzing Stream Reasoning

Thomas Eiter
Knowledge Based Systems Group, TU Wien

The rise of smart applications has drawn interest to logical reasoning over data streams. Recently, different query languages and engines for stream processing respectively stream reasoning were proposed in different communities. However, due to a lack of theoretical foundations, the expressiveness and semantics of diverse approaches was given only informally, and their semantic relationship. Towards clear specifications and means for analytic study, a formal framework is desired that allows to characterize their semantics in precise terms. Inspired by this, we develop a logic-based such framework, which features window operators that provide a flexible mechanism to represent views on streaming data and temporal operators over windows, which also may be nested. In addition, we define a rule language on top. We present the emerging formalism on examples and discuss some complexity issues for it. Furthermore, we briefly consider its usage and relationship to stream query respectively streams reasoning languages, in particular to the Continuous Query Language (CQL) and ETALIS. We finally address recent results and ongoing research around LARS, which is carried out in a project funded by the Austrian Science Fund.

Answer Set Solving in a nutshell

Torsten Schaub
University Potsdam

The tutorial aims at acquainting the participant with ASP's modeling and solving methodology, enabling her/him to conduct independent problem solving using ASP systems. To this end, the tutorial starts with a brief introduction to the essential formal concepts of ASP, needed for understanding its semantics and solving technology. In fact, ASP solving rests on two major components: A grounder turning specifications in ASP's modeling language into propositional logic programs and a solver computing a requested number of answer sets of the program. We illustrate both ASP's grounding techniques and the basic ideas of the underlying solving technology. Finally, we sketch the usage of ASP in conjunction with Python for modeling complex reasoning scenarios. This involves an introduction to the API of clingo 4, an ASP system with control capacities expressible in Python. We illustrate this by developing the board game of Ricochet Robots.

All involved ASP systems are freely available from http://potassco.sourceforge.net.

Complexity in Logic: off the beaten track towards pastures new

Uli Sattler
University of Manchester

Research in logic is hard: first, there are loads of existing formalisms, their relationships, model properties, computational complexity classes, etc. that we have to learn about and understand. Then, there is a wide range of skills to acquire, including writing, note keeping, proof techniques, and analytical thinking. Of course, we need to narrow down an interesting yet solvable research question and make progress towards its solution. Many of these questions are basically "what is the computational complexity of problem X in logic Y?". In my tutorial, I will first describe the above sources of complexity in more detail, and then present some interesting cases where starting from but straying from this "beaten track" of computational complexity has resulted in interesting insights. These cases include new reasoning problems or tasks, and also alternative measures of complexity.

New directions in typelogical semantics

Michael Moortgat
Universiteit Utrecht

Typelogical grammars are substructural logics that provide a prooftheoretic perspective on natural language syntax and semantics. In this tutorial I discuss two recent developments in this area. The first (see [1] and much subsequent work) is the integration of Montague-style compositional interpretation with a vector-based account of lexical meanings. The second development ([2] and subsequent work) systematically extends the vocabulary of Lambek's original calculus: next to composition ('merge') and its residuals, one adds decomposition and subtraction, and a set of unary operators (related to the '!' of linear logic) licensing limited forms of reordering/restructuring that leave the form-meaning correspondence intact. I will discuss the connection between these two developments and the opportunities for transfer of results from one to the other.

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Cut-elimination in generalisations of the sequent calculus

Revantha Ramanayake
TU Wien

Logic is concerned with the study and use of valid reasoning. The most well-known logics are classical propositional and first-order logic. Nevertheless, various other forms of reasoning are needed to model the different applications and situations that arise in practice, giving rise to many new logics ('non-classical logics') that are more expressive and permit finer distinctions than classical logic. The study of such logics is interesting in its own right and also yields an understanding of the systems modelled by these logics.

The basis of the proof-theoretical approach to studying a logic is the notion of proof, and in particular, the study of the formal proof systems of the logic (a statement belongs to the logic if and only if it has a proof in such a proof system). Gentzen introduced the formal proof-system called the sequent calculus in order to study the structure of proofs in classical and intuitionistic logic. His celebrated cut-elimination theorem implies the sub-formula property which states that a statement can be proved in the calculus using only sub-formulae of the formulae occurring in the statement. The point is that the proofs from his calculus have a nice normal form (such calculi are called analytic). Contrast, for example, with proofs in the Hilbert calculus where formulae that occur in the proof might not occur in the statement that is proved. Gentzen made use of his result to give a formal proof of consistency of Peano arithmetic using a suitable induction principle.

Note that if we are not interested in having a calculus with the subformula property, then it is easy to obtain a sequent calculus for most logics of interest. However the key to a proof-theoretical study of the logic is an analytic calculus. For many logics of interest it is not clear at all how to obtain an analytic sequent calculus. This has led to the introduction of various generalisations and extensions of the sequent calculus. These generalisations yield analytic calculi for some logics and not for others and it is still not clear why cut-elimination seems to fail for certain logics in certain formal systems (negative results stating that a given formal system cannot yield an analytic calculus for a certain logic are rare!).

In this tutorial I will discuss two generalisations of the sequent calculus: the hypersequent calculus and the Display Calculus. I will pay special attention to the display calculus which has been used to provide analytic formal systems for many different non-classical logics. An attractive feature of the display calculus is the natural general cut-elimination theorem which applies whenever the rules of the calculus satisfy certain easy to check conditions. I will ‘take apart’ the display calculus in order to motivate why the rules of calculus take the shape that they do, and why this leads to a general cut-elimination theorem. I will then explain how a display calculus for a logic can be extended by suitable new rules to obtain a display calculi for axiomatic extensions of that logic. The preservation of analyticity under the addition of new rules is known as modularity and this property usually fails in the sequent calculus but holds for the rules of the display calculus we introduce here. An intention of this tutorial is that the insights on proof systems and cut-elimination explained via the display calculus will be applicable to various other proof systems as well.

Winners of the VCLA International Student Awards

Boolean Circuit Optimization

Sophie Spirkl
Princeton University

We consider the problem of constructing adders with prescribed input arrival times. Most previous results implement parallel prefix graphs (e.g. Kogge-Stone) and are designed for uniform input arrival times. We generalize the concept of prefix graphs, which allows us to reduce single-output adder optimization problems to a tree structure, and we allow arbitrary input arrival times. For both single-output and full adders, we present efficient algorithms which construct adders that improve, even for uniform arrival times, upon previous results in the core objectives delay, size and fan-out.

Deciding Properties of Automatic Sequences

Luke Schaeffer
MIT

Automatic sequences are self-similar sequences, defined in terms of finite automata, which arise naturally as the solutions to pattern avoidance problems. We show that several questions about automatic sequences can be expressed as logical predicates in a decidable first-order theory, and then answered purely mechanically by a decision algorithm. With the aid of this algorithm, we recover a surprising number of known results, and prove interesting new theorems as well. We also improve the theoretical power of this approach by extending the logical theory to broader classes of sequences, adding new operations, and showing how to better interpret the results.

SAT Sampling: From Theory to Practice

Kuldeep S. Meel
Rice University

Counting the number of satisfying truth assignments of a given Boolean formula or sampling such assignments uniformly at random are fundamental computational problems in computer science with numerous applications. In computer-aided design, these problems come up in constrained-random verification, where test input vectors are described by means of constraints. While the theory of these problems has been thoroughly investigated in the 1980s, approximation algorithms developed by theoreticians do not scale up to industrial-sized instances. Algorithms used by the industry offer better scalability, but give up certain correctness guarantees to achieve scalability. We describe a novel approach, based on universal hashing and SMT, that scales to formulas with hundreds of thousands of variables without giving up correctness guarantees.

New Complexity Bounds for Evaluating CRPQs with Path Comparisons

Pablo Munoz
University of Chile

Graph databases make use of logics that combine traditional first order features with navigation on paths, in the same way logics for model checking do. However, modern applications of graph databases impose a new requirement on the expressiveness of the logics: they need comparing labels of paths based on word relations. This has led to the study of logics that extend basic graph languages with features for comparing labels of paths based on regular relations, or the strictly more powerful rational relations. The evaluation problem for the former logic is decidable (and even tractable in data complexity), but already extending this logic with such a common rational relation as subword or suffix turns evaluation undecidable. We thus study less expressive logics that still allow comparing paths based on practically motivated rational relations. Here we concentrate on the most basic such languages, which extend graph pattern logics with path comparisons based only on suffix, subword or subsequence. The results provide a complete landscape of the complexity of evaluation for each one of these logics, which are all decidable in elementary time. The extension with suffix is even tractable in data complexity, while the other two are not. To obtain our results we establish a link between the evaluation problem for graph logics and two important problems in word combinatorics: word equations with regular constraints and square unshuffling.

Contributed Talks

Intertranslatability of Abstract Argumentation Frameworks

Sylwia Polberg

Over the last years, argumentation has become an influential field in Artificial Intelligence [1]. One of its subfields is *abstract argumentation*, at the heart of which lies the abstract argumentation framework (AF) developed by Phan Minh Dung [2]. Although well acknowledged, AFs have their shortcomings, which inspired a search for more general models [3]. Throughout the years, many AF extensions were created, ranging from the ones employing various values and preferences [4, 5, 6] to those that focus on researching new types of relations between arguments [7, 8, 9, 10, 11, 12]. This amount of frameworks should not come as a surprise. Argumentation is a wide area with numerous applications, in which one has to face different classes of problems. Frameworks of a given type can be seen as tools to model particular issues and concepts, which on one hand gives us more insight into how to approach the problems, but on the other affects the framework's design. When facing such amount of available structures, it is only natural to ask whether one can translate one framework into another, particularly in a way that preserves the desired semantics, and what the consequences of such a process are.

Framework intertranslatability is an interesting topic from both the practical and theoretical point of view. When it comes to applications, establishing a transformation can be useful for the design of argumentation–based software. While modeling problems in more advanced frameworks has its benefits, many structures do not have dedicated solvers. Consequently, translating a framework into one that is supported by a software implementation is of practical value. Moreover, based on available translations and their complexity, we can choose structures optimal for further solver development. Finally, if a translation of a given framework produces a particular subclass of the target structure, we can use this knowledge to further improve the targets solver.

Translations are often used to compare the expressive power of given formalisms. While in the application–oriented approach we first and foremost look for an approach that would produce us the desired extensions, here we search for transformations that exhibit particular properties such as faithfulness, polynomiality and modularity [13]. Additionally, in case we cannot find any, we try to establish impossibility results proving that they in fact cannot exist. However, our research interests go beyond the expressive power. First of all, we search for "generic" translations, i.e. ones that are as independent from the argumentation semantics as possible. Although dedicated transformations can use various properties of given semantics to e.g. boost efficiency, generic ones are more useful in exposing and comparing the design choices and capabilities of the frameworks we are interested in. Furthermore, we want to establish the connection between various relations between arguments, in particular between different forms of support and attack. We would like to know if it is possible to transform one into another and what is lost or gained in the process, thus providing an answer to the question what is the value of researching additional relation types in abstract argumentation frameworks.

In our study we introduce a wide range of translations between the aforementioned frameworks that research new relations between arguments [2, 7, 8, 9, 10, 11, 12]. Some methods have already been researched, especially transformations from and to the Dung framework. We extend these results by providing a way to transform the abstract dialectical framework into the Dung framework which so far has been done for only several semantics [14]. Furthermore, the existing methods for frameworks with support [15, 16, 10] can be classified as coalition approaches, i.e. the arguments in the target structure represent sets of arguments of the source structure that are connected by support. We propose two alternative transformations – the defender and attack propagation – inspired by the research in [17, 8], which aim to simulate the behavior of support with combinations of attack and defense. Finally, despite the available research, there are less results concerning moving between AF generalizations [8]. Consequently, we introduce new approaches and follow up on our previous work [11].

We also provide an in–depth analysis of our translations in terms of functional, complexity, syntactical and semantical properties. The first group studies the translation as a function and answers questions such as: can all source frameworks be translated, can any target framework be obtained, and whether there is a one–to–one relation between them. The complexity properties focus on how difficult the translation is in terms of computational time, modularity, required semantical knowledge of a given framework and the size of the translated structure relative to the source one. Syntactical properties tell us about the difference in the domains of arguments and introduction or removal of arguments and relations between the source and target frameworks. Finally, the last group compares the semantics behavior of both structures, i.e. if all the desired extensions and only them can be obtained, are auxiliary arguments required and whether the translation is specialized for particular semantics or is generic and can be applied in more cases. This analysis allows us to separate available argumentation frameworks into groups, inside of which we can "move" between structures at a low cost. This separation can be seen as a rough classification of expressive power of available argumentation frameworks.

To summarize, we introduce a wide range of translations between argumentation frameworks and analyze their properties in order to compare and analyze the frameworks. However, the purpose of our research is not to advocate the use of one relation or framework above the other. By the means of translations we want to show the consequences of changing between different structures with different types and amounts of elements. The consequences which, depending on the problems we want to model, can be considered very heavy or completely negligible. Therefore, our classification and comparison can be seen as an aid in choosing tools adequate for our purposes, while translations can be used to limit some of their possible drawbacks.

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Combining Different Forms of Defeasible Reasoning in Abstract Argumentation: Integrating Mechanisms from Adaptive Logics

Jesse Heyninck

In this talk I will present a framework that allows for the combination of different forms of defeasible reasoning (in short, DR).

DR (e.g. inconsistency handling, inductive generalizations, abduction, normative reasoning, etc.) is an essential form of reasoning whenever encountering complex, uncertain or incomplete information. When we reason defeasibly, we draw conclusions only tentatively, i.e. they may be retracted in the light of new information. This means that the support of a defeasible argument is not deductive: it is possible that the premises of a valid defeasible argument are true while its conclusion is false. The conclusion is thus drawn conditionally, and when the assumptions under which the conclusion is drawn turn out to be dubious, we retract this inference. We can thus say that DR always consists of an ampliative aspect (i.e. the fact that there is more derivable then what is deductively guaranteed) and a corrective aspect (i.e. the fact that conclusions can be retracted if they turn out to be based on dubious assumptions). This also makes clear that DR has an essentially dialectical character: defeasible inferences are assumed valid until and unless encountering an argument against their validity.

A formal model that allows for the systematic study of the dialectical character of DR is abstract argumentation, developed in the seminal paper by Dung [5]. In this framework arguments are arranged in directed graphs $\langle \mathcal{A}, \rightarrow \rangle$ in which $\mathcal{A}$ is a set of arguments and $\rightarrow \subseteq \mathcal{A} \times \mathcal{A}$ represents argumentative attacks (e.g. Pollock's rebuttals and undercuts). Given such a graph, argumentation semantics specify criteria for selecting sets of arguments that represent stances of rational discussants. In recent years, however, abstract argumentation has received some criticism for lacking an account of the logical structure of argumentation (see for example [3] and [10]). Not only is the logical structure of argumentation an essential part of the practice of reasoning, there are also some more specific problems caused by the high level of abstraction on which abstract argumentation operates. For example, abstract argumentation semantics in general fail to fulfil certain intuitive rationality constraints such as the consistency or the logical closure of selected conclusions. Several proposals to add more structure to AFs have been made in the literature (see for example: [1], [9], [8], [7], [6]). One problem of these approaches is, however, that within the logical models they depend on, the mechanisms of specific instances of DR frequently used in science and the interplay of different forms of DR has not been studied in detail.

A good starting point to improve on these approaches is to integrate mechanisms from a formal framework within which many different DR forms have been modelled. Such a framework is given by adaptive logics. The standard format for adaptive logics [2, 11] offers a generic framework for DR. Its dynamic proof theory extends a monotonic, reflexive and transitive core logic (**L**) with a set of retractable inferences which are associated with defeasible assumptions. More specifically, these assumptions are sets of formulas Δ of a predefined 'abnormal' form that are assumed to be false in the given inference. When an assumption turns out to be dubious in view of a premise set Γ, e.g. when some $A \in \Delta$ is derived from Γ in **L** as part of a minimal disjunction of abnormalities, the inference associated with it gets retracted. Various adaptive strategies offer mechanisms for this retraction of inferences, some following a more cautions rationale then others. A plethora of forms of defeasible reasoning has been explicated in the adaptive logic framework.

Examples are inconsistency-adaptive logics based on the paraconsistent core logic **CLuN** where abnormalities are contradictions $A \wedge \neg A$. One retractable inference rule is disjunctive syllogism (DS): $\neg A, A \vee B$ implies B on the assumption that $A \wedge \neg A$ is false. Take the premise set $\Gamma = \{\neg p, p, \neg r, p \vee s, r \vee q\}$. While applying DS to $\neg r$ and $r \vee q$, assuming that $r \wedge \neg r$ is false, is not retracted, applying DS to $\neg p$ and $p \vee s$ will be retracted since $p \wedge \neg p$ is derivable.

In many real life examples of DR, different forms of DR are used in combination with one another. Only recently have the first steps been taken in the direction of a framework that allows for the combination of different sorts of adaptive logics (see [12], [13], [11, Ch.5] and [4]). In contrast to the standard format of adaptive logics, in these approaches there is no transparent account of the dialectical character of defeasible inferences. Also, the field of adaptive logics still stands in relative isolation from approaches to defeasible reasoning in the argumentative tradition, such as **ASPIC** [9], Defeasible Logic Programming [7] and argumentation based on classical logic [6]. This isolation inhibits both the comparative studies and the cross-fertilization between these two approaches.

In this talk I present a framework for the representation of combinations of DR forms in formal argumentation. In this framework, arguments are generated by (possibly multiple) stable core logics and the retractable inference rules extending them. Argumentative attacks are then defined to represent the mechanism that retracts inferences based on dubious assumptions. This framework allows us to integrate the well-known mechanisms of **ALs** for the modelling of DR forms . It retains a clear account of the dialectical nature of DR without excluding expressiveness. I give representation results for the standard, lexicographic and colexicographic format of **AL**. Furthermore, I give a representation result for **ASPIC**, one of the best known frameworks for structured argumentation. The representational results of the lexicographic and colexicographic format of adaptive logic are used to show the potential to model combinations of DR forms. I end the talk by pointing to promising future research based on this framework that could be beneficial both to the adaptive logic and the formal argumentation communities.

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An Abstract Algebraic Logic view of propositional-attitude aggregation theory

María Esteban and Zhiguang Zhao

The theory of *social choice* is the formal study of mechanisms for collective decision making, and investigates issues of philosophical, economic, and political significance, stemming from the classical Arrovian problem of how the preferences of the members of a group can be "democratically" aggregated into one outcome.

In the last decades, many results appeared generalizing the original Arrovian problem (e.g. abstract and algebraic aggregation theory by Wilson [16] and Rubinstein and Fishburn [15]), which gave rise to a research area called *judgment aggregation* (JA) [13], which studies how a group of individuals aggregate their individual judgments on logically interconnected propositions (the *agenda*) into collective judgments on them. While the original work of Arrow [1] focuses on preference aggregation, this can be recognized as a special instance of a consistent judgments, expressed by each member of a group of individuals over a given set of logically interconnected propositions: each proposition in the agenda is either accepted or rejected by each group member, so as to satisfy certain requirements of logical consistency. Within the JA framework, the Arrovian-type *impossibility results* (providing sufficient axioms for aggregator functions to turn into degenerate rules, such as dictatorship) are obtained as consequences of *characterization theorems* [14], which provide necessary and sufficient conditions for agendas to have aggregator functions on them satisfying given axiomatic conditions.

In the same logical vein, in [12], *attitude aggregation theory* was introduced; this direction has been further pursued by Herzberg [8], where a characterization theorem has been given for certain many-valued propositional-attitude aggregators as MV-algebra homomorphisms, and a well-known result from judgment aggregation has been derived from it as corollary.

Methodologically, the *ultrafilter argument* (see Kirman and Sondermann [10], Lauwers and van Liedekerke [11]) is the tool, derived from logic, underlying the generalizations and unifications mentioned above. It can be sketched as follows: to prove impossibility theorems for finite electorates, one shows that the axiomatic conditions on the aggregation function force the set of all decisive coalitions to be an (ultra)filter on the powerset of the electorate. If the electorate is finite, this implies that all the decisive coalitions must contain one and the same (singleton) coalition: the oligarchs (the dictator). First employed in the proof of Arrow's theorem, this argument was applied to obtain elegant and concise proofs of impossibility theorems in judgment aggregation [3]. More recently, it gave rise to characterization theorems, e.g. establishing a bijective correspondence between Arrovian aggregation rules and ultrafilters on the set of individuals [9]. Moreover, by means of the well-known correspondence between ultrafilters and Boolean homomorphism, such argument has been generalized by Herzberg to get a bijective correspondence between certain judgment aggregation functions and ultraproducts of profiles [7].

The present paper is motivated by the observation that *Abstract Algebraic Logic* (AAL) is the natural theoretical setting for Herzberg's results.

Abstract Algebraic Logic (AAL) is a forty-years old research field in mathematical logic (see [5], [6]). It was conceived as the framework for an algebraic approach to the investigation of logics: its main goal was establishing a notion of *canonical algebraic semantics* uniformly holding for classes of logics, and using it to systematically investigate properties of logics in connection with properties of their algebraic counterparts.

In the proposed talk, we report the results in the working paper [4], in which an Abstract Algebraic Logic perspective is given on propositional attitude aggregation theory. Specifically, we generalize and refine Herzberg's result in [8] from the MV-algebra setting to any class of algebras canonically associated with some *selfextensional logic*. This notion encompasses a vast class of logics, of which classical, intuitionistic, modal, many-valued and relevance logic are special cases.

In particular, we improve Herzberg's characterization result which consisted of two slightly asymmetric parts. The present characterization result is symmetric, and holds for the class of algebras canonically associated with any selfextensional logic $\mathcal{S}$. Aggregation of propositional attitudes modeled in classical, intuitionistic, modal, many-valued and relevance logic can be uniformly captured as special cases of the present result. This result paves the way to the systematic study of a wide array of "realistic agendas" of formulas the propositional connectives of which are interpreted in ways which depart from the classical interpretation. Conceptually, adopting nonclassical logics to formalize natural language statements makes it possible to more adequately capture the original meaning in different situations. Technically, the nonclassical interpretation of the logical connectives is useful in the light of the fact that, as observed by Dietrich [2], it can provide a strategy to escape the impossibility results (for instance, in the case of [2], Dietrich escaped the impossibility results by interpreting conditional propositions of the form "if a then b" by means of subjunctive implication rather than classical implication).

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Basic model theory of modal languages with propositional constants

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1 Introduction

From a semantic perspective a propositional modal language $\mathcal{L}_{MOD}$ can be viewed either as a fragment of a first-order language $\mathcal{L}_1$, when interpreted in models, or as a fragment of a monadic second-order language $\mathcal{L}_2$, when interpreted in frames [5]. Here I will examine a modal language enriched with a set of *propositional constants* $CON = \{c_1, c_2, ...\}$, which are symbols for special propositions, each introduced with a set of characteristic axioms. These axioms define the meaning of propositional constants, i.e. are used to restrict the set of their possible interpretations in a frame and are not closed under uniform substitution. Furthermore, axioms for constants can be expressed either in the modal language itself or in a metalanguage, for instance $\mathcal{L}_1$ or $\mathcal{L}_2$; I will be concerned only with a language where all constants are "internally axiomatized" (i.e. within the modal language), which can be called $\mathcal{L}^c_{MOD}$. Some examples of modal languages enriched with propositional constants of various kind may be found in [1], [2], [3] and [4].

2 Frames and models

$\mathcal{L}^c_{MOD}$-formulas are interpreted in *general frames with specific restrictions*. A general frame with specific restrictions is $\mathcal{G} = \langle W, R, \Pi, \{\Pi(c_i) | c_i \in CON\}\rangle$, where $\Pi \subseteq 2^W$ is the set of admissible interpretations for propositional variables (the restriction for propositional variables) and $\Pi(c_i) \subseteq \Pi$ is the set of admissible interpretations for the i-th constant in the language (the restriction for the i-th constant). Notice that $\Pi(c_i)$ is not required to be closed under boolean and modal operations. An *admissible model* on these frames is $\mathcal{M} = \langle \mathcal{G}, V\rangle$, where V is a valuation function such that $V(p_j) \in \Pi$ for any variable p_j and $V(c_i) \in \Pi(c_i)$ for any constant c_i. Satisfiability and validity of $\mathcal{L}^c_{MOD}$-formulas in admissible models and frames are defined accordingly.

3 The proper range of a formula

The notion of *proper range* will be used to distinguish a certain class of models for a formula. Let C_k be the class of all general frames (with specific restrictions) whose domain has cardinality k. We say that $\mathcal{G} = \langle W, R, \Pi, \{\Pi(c_i)|c_i \in CON\}\rangle \in C_k$ has the *weakest restrictions of level k* for an $\mathcal{L}^c_{MOD}$-formula ψ when:[1]

- $\mathcal{G} \vDash \psi$;
- $|\Pi| \geq |\Pi'|$, for any restriction Π' s.t. there is $\mathcal{G}'[\Pi'] \in C_k$ and $\mathcal{G}'[\Pi'] \vDash \psi$;
- for any $\Pi' \supset \Pi$, there is no $\mathcal{G}'[\Pi'] \in C_k$ s.t. $\mathcal{G}'[\Pi'] \vDash \psi$;
- for any constant $c_i \in \psi$, $|\Pi(c_i)| \geq |\Pi'(c_i)|$, for any specific restriction $\Pi'(c_i)$ s.t. there is $\mathcal{G}'[\Pi'(c_i)] \in C_k$ and $\mathcal{G}'[\Pi'(c_i)] \vDash \psi$;
- for any $\Pi'(c_i) \supset \Pi(c_i)$, there is no $\mathcal{G}'[\Pi'(c_i)] \in C_k$ s.t. $\mathcal{G}'[\Pi'(c_i)] \vDash \psi$.

The proper range of ψ is defined as the smallest class including:

- for any cardinal k, all general frames with the weakest restrictions of level k for ψ;
- the disjoint union of any family of frames already in the proper range of ψ.

This definition has some relevant analogous for languages without constants: indeed, it can be proved that the proper range of an $\mathcal{L}_{MOD}$-formula ϕ corresponds to the class of all models based on Kripke frames where ϕ is valid, or, equivalently, to the class of all models built on general frames where ϕ is valid and $\Pi = 2^W$.

4 Invariance and preservation results

It is well-known that the satisfiability of an $\mathcal{L}_{MOD}$-formula in a model is invariant under generated submodels, disjoint unions, p-morphic images and ultrafilter extensions, whereas the validity of an $\mathcal{L}_{MOD}$-formula in a frame is preserved under generated subframes, disjoint unions and p-morphic images and anti-preserved under ultrafilter extensions [5]. It can be proved that for $\mathcal{L}^c_{MOD}$-formulas the situation is analogous. However, the expressivity of $\mathcal{L}_{MOD}$ and $\mathcal{L}^c_{MOD}$ is different; for instance, there is an $\mathcal{L}^c_{MOD}$-formula ϕ containing two inverse operators of necessity and a constant c which has for its proper range a class of irreflexive general frames. In particular, we will see that:

- ϕ is valid in the general frame $\mathcal{Z} = \langle \mathbb{Z}, <, \Pi, \Pi(c)\rangle$, where $\mathbb{Z}$ is the set of integers, $<$ is the relation "smaller than", $\Pi = 2^{\mathbb{Z}}$ and $\Pi(c) = \{\{n\}|n \in \mathbb{Z}\}$;
- there is no p-morphic image of $\mathcal{Z}$ containing a reflexive point.

[1] Expressions like $\mathcal{G}[\Pi]$ are shorthands to say that Π is the restriction in $\mathcal{G}$.

$\mathcal{L}^c_{MOD}$ differs in expressivity also from a modal language with nominals; for instance, in [2] it is proved that the validity in a frame of a modal formula containing nominals is not preserved under disjoint unions and p-morphic images. The reason is that nominals can be seen as a particular kind of propositional constants which are not "internally axiomatized", since some of their intended properties need to be expressed in $\mathcal{L}_1$ or $\mathcal{L}_2$.

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Modal Logics for Qualitative Spatial Reasoning

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Space is an inexhaustible source of inspiration that for ages has fascinated mathematicians, logicians and philosophers, among others. In this note, we are mainly focused on formal methods for space representation and reasoning. Such formalisms have been recently thoroughly studied by researchers from the fields of Artificial Intelligence (AI) and Knowledge Representation (KR), as we can find out from the preface of [1]:

> 'Space, with its manifold layers of structure, has been an inexhaustible source of intellectual fascination since Antiquity. [...] In this long intellectual history, however, one relatively recent, yet crucial, event stands out: the rise of the logical stance in geometry. Fundamental to this development is the analysis of geometrical structures in relation to the formal languages used to describe them, and the recognition of the special mathematical challenges – and opportunities – which such an analysis presents.'

What is interesting from the cognitive and philosophical points of view is that humans possess the abilities to reason about space that can hardly be obtained by means of any AI system. Until now, no AI method has been able to perform as precise and universal spatial reasoning as humans. This situation motivates us to work on spatial reasoning methods that would imitate human-like reasoning aspects. In particular, human-like methods seem to be based on qualitative approach being in opposition to quantitative (numerical) approach. We are convinced that using qualitative methods will be an important step towards better understanding of human spatial reasoning methods. Additionally, this approach is usually faster, has lower computational complexity and is easier to understand for humans than the quantitative approach. As a result, the qualitative approach has a number of practical spatial applications, e.g., in geographical information systems [2] or in robotics [3].

There are three main formal approaches to qualitative spatial reasoning, namely relational algebras, first-order theories and modal logics. Relational algebras (e.g., see [9]) deal with objects and binary, jointly exhaustive and mutually disjoint relations between them. These purely existential theories are formulated as constraint-satisfaction systems, i.e., they check whether a given set of objects and relations between them is consistent. Within the second approach (e.g., see [8]) various first-order spatial theories of such spatial concepts as topology or directional information are considered. Finally, there are spatial modal logics (e.g., see [4]) that introduce modalities to represent spatial relations between objects. The last approach makes it possible to obtain very expressive systems, but in many cases they have high computational complexity (often they are even undecidable).

The first part of the presentation introduces the motivation for our work on spatial formalisms. Afterwards, we confine ourselves to the description of qualitative spatial modal logics and in particular, to two-dimensional logics for directional information representation. We present Compass Logic [10], Spatial Propositional Neighborhood Logic (SpPNL) [7] and Cone Logic [6]. The abovementioned logics use the two-dimensional Cartesian coordinate system in which a point is identified with a pair of numbers, namely its x and y coordinates. In this environment, various modal operators that represent directional relations between points (or regions in the case of SpPNL) are introduced.

- Compass Logic [10] involves two irreflexive linear orders for two Cartesian coordinates. The first of them is to be interpreted as "lying horizontally" and the second as "lying vertically" on a Cartesian grid. Four modalities are introduced that enable us to move along one of the Cartesian axes keeping the other coordinate constant. In other words, the modalities provide an access to points lying on the same horizontal (or vertical) line as the given point. The logic is proven to be undecidable [5].

- Spatial Propositional Neighborhood Logic [7] is another logic with projection modalities that uses two linear orders for Cartesian coordinates. However, instead of points (as in Compass Logic), SpPNL considers two-dimensional regions. Four modalities are introduced which enable us to move along x and y axes of the Cartesian coordinate system. SpPNL is also undecidable.

- Cone Logic [6] is not a projection-based spatial logic like the two previously mentioned systems. Instead, it is based on the cone-shaped cardinal directions. Given a particular point, it divides a plane into four regions: lower-left, lower-right, upper-left and upper-right quadrants. Afterwards, eight modalities are introduced that make it possible to access points located in these quadrants. Four of the modalities treat quadrants as open regions, whereas other four treat quadrants as semi-closed regions. The logic is decidable and proven to be PSpace-complete.

The abovementioned, well-known directional modal logics capture space from the aerial point of view. This way of space description is appropriate, e.g., for maps' representation but it does not reflect the subject's perspective and the way she perceives the surrounding space. On the other hand, the subject-oriented representation may be used as an interface between machine and humans in a sense that qualitative spatial information expressed by a human would be interpreted by a machine and the other way round. As a result, such interfaces may be applied in systems that describe surrounding space and navigate a blind person. Systems with subject-oriented representation may also be used in mobile robotics for autonomous navigation. In order to capture the subject-oriented representation we propose a novel modal language called $\mathcal{HLQL}$. We present a new type of directional operators that represent information about object's position with respect to another object from a subject's point of view. These operators involve 3 elements (in standard approaches only 2 elements are considered) and enable us to represent such relations as: "from my current point of view, the post office is to the left of the church". Technically,

- $\mathcal{HLQL}$ is a basic hybrid multi-modal language (involving nominals and satisfaction operators) augmented with appropriately tailored accessibility relations and the constant symbol s (for the subject). The semantics for the logic is Kripke-structure based: a frame is a plane, either finite or infinite, with polar coordinates, divided into cells of arbitrary length and angle-width. The central locus is occupied by the subject. We introduce modal operators that make it possible to express directional position of an object with respect to the subject, an object's location with respect to another object, seen from the subject's perspective, and also qualitative distance relations between objects. We have proven that the axiomatization of $\mathcal{HLQL}$ over finite domains is sound and complete with respect to the indicated class of structures. For finite domains the logic is decidable.

In our future work we would like to devise a tableau system for $\mathcal{HLQL}$. Furthermore, we will work on possible applications of the logic, e.g., to the mobile robot's navigation or to assistance to a blind person.

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Algebra-based canonical formulas for superintuitionistic logics

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Superintuitionistic logics (si-logics for short) are propositional logics extending intuitionistic propositional calculus (**IPC**). Finding uniform axiomatizations for si-logics has been a significant problem in this area. Axiomatization methods via the so-called frame-based formulas were studied by Jankov, de Jongh, Fine and later by Zakharyaschev (see e.g. [4] for an overview). Extending the approaches of Jankov, de Jongh and Fine, Zakharyaschev defined canonical formulas that axiomatize all si-logics. Among other things Zakharyaschev's canonical formulas provide a transparent alternative proof of the Blok-Esakia theorem. In addition, canonical formulas give rise to particular classes of si-logics. For instance, the well-known classes of subframe and cofinal subframe logics are exactly the logics axiomatizable by canonical formulas where a specific parameter is restricted. Zakharyaschev also defined canonical formulas for modal logics that axiomatize all normal extensions of **K4**. Jeřábek [5] generalized canonical formulas to canonical multi-conclusion rules that axiomatize modal and intuitionistic multi-conclusion consequence relations.

Zakharyaschev's method is model-theoretic in nature. Recently, an algebra-based approach to canonical formulas for si-logics has been developed in [1, 2]. The algebraic perspective reveals that the main technical tool used in the theory of canonical formulas is the disjunction-free reduct of Heyting algebras. This reduct is locally finite[1], which is the key property for this method. This insight promises generalizations of the method of algebra-based canonical formulas to other non-classical logics that have algebraic semantics. In this talk we provide an overview on this algebra-based approach to canonical formulas for si-logics and point towards future research using this method.

The algebraic counterpart of Zakharyaschev's canonical formulas for si-logics are called $(\wedge, \to)$-canonical formulas. The $(\wedge, \to)$-canonical formulas fully describe the disjunction-free reduct of a finite subdirectly irreducible (s.i.) Heyting algebra A, but describe the structure of the missing connective $\vee$ only partially on a subset $D \subseteq A^2$. The $(\wedge, \to)$-canonical formulas, though syntactically quite different, serve the same purpose as Zakharyaschev's formulas, namely providing uniform axiomatizations of

[1]Recall that a variety V is locally finite iff every finitely generated algebra $A \in V$ is finite. It is well known that the variety of Heyting algebras is not locally finite. In fact, already the free Heyting algebra with one generator, the so-called Rieger-Nishimura lattice, is infinite.

all si-logics. In fact, any given axiomatization of some si-logic can be transformed effectively into an axiomatization of the logic in terms of $(\wedge, \to)$-canonical formulas.

In analogy with Zakharyaschev's canonical formulas, $(\wedge, \to)$-canonical formulas give rise to subframe and cofinal subframe logics. These logics are axiomatized by $(\wedge, \to)$-canonical formulas where $D = \emptyset$ and depending on whether the behavior of $\neg$ is taken into account. Subframe and cofinal subframe logics are well-studied large classes of si-logics with good properties such as the finite model property (FMP). In algebraic terms, a logic is a subframe logic if its corresponding variety of Heyting algebras is closed under $(\wedge, \to)$-subalgebras and a cofinal subframe logic if it is closed under $(\wedge, \to 0)$-subalgebras.

The proof method used to establish the theory of $(\wedge, \to)$-canonical formulas relies on the locally finiteness of the disjunction-free reduct of Heyting algebras. Besides the $(\wedge, \to)$-reduct, Heyting algebras have other well-known locally finite reducts. As a matter of fact, some of these are suitable candidates for different kinds of algebra-based canonical formulas.

Indeed, in [2] canonical formulas based on the bounded lattice reduct of Heyting algebras were introduced. These formulas were called $(\wedge, \vee)$-canonical formulas. They fully describe the bounded lattice structure of a finite s.i. Heyting algebra A and only partially the $\to$-structure on the domain prescribed by an additional parameter $D \subseteq A^2$. As in the $(\wedge, \to)$-case, the $(\wedge, \vee)$-canonical formulas axiomatize all si-logics. Neglecting the parameter D in $(\wedge, \vee)$-canonical formulas leads to stable si-logics that form the $(\wedge, \vee)$-analogue of subframe logics. Stable logics form a well-behaved class of logics, in particular, they have the finite model property. In fact, many well-known si-logics are stable and in [2] it was shown that there is a continuum of stable logics.

In this talk, we will provide an overview of the algebra-based approach to canonical formulas. We will focus on algebra-based formulas for si-logics. In particular, we will sketch the algebraic proof of the main result of canonical formulas stating that all si-logics are axiomatizable by these formulas. Moreover, we will give examples of si-logics axiomatized by particular canonical formulas. We will also present new results on cofinal stable logics that form the $(\wedge, \vee)$-analogue to cofinal subframe logics (see [3] for details). Finally, we will discuss how the method of algebra-based canonical formulas can be applied to other non-classical logics. Indeed, it seems to be a major feature of this axiomatization method that it can be applied to many classes of non-classical propositional logics with suitable algebraic semantics. This has been done for the case of modal logics. But the method promises to be applicable to wider classes of logics such as substructural logics[2], or multi-modal logics such as **PDL**. We will discuss ongoing work in these directions.

[2]As a start, canonical formulas for the k-potent fragment of Lambek calculus have been presented recently at LATD 2014 by N. Bezhanishvili, N. Galatos and L. Spada.

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Towards Reconciling SPARQL and Certain Answers

Shqiponja Ahmetaj and Wolfgang Fischl

In the recently released recommendation [6], the W3C has defined various SPARQL entailment regimes to allow users to specify implicit knowledge about the vocabulary in an RDF graph. The theoretical underpinning to the systems for query answering under rich entailment regimes is provided by the big body of work on ontology-based query answering, notably in the area of Description Logics (DLs) [3]. However, the semantics of query answering under SPARQL entailment regimes is defined in a more naive and much less expressive way than the *certain answer semantics* usually adopted in the DL and database literature.

Example 1. Consider an RDF graph G containing a single triple $(b, \mathsf{a}, \mathsf{Prof})$ – stating that b is a professor – and an ontology $\mathcal{O}$ containing the triples

$$(\mathsf{Prof}, \texttt{rdfs:sc}, \texttt{\_:b}), (\texttt{\_:b}, \mathsf{a}, \texttt{owl:Restriction}),$$
$$(\texttt{\_:b}, \texttt{owl:onProperty}, \mathsf{teaches}), (\texttt{\_:b}, \texttt{owl:someValuesFrom}, \texttt{owl:Thing}).$$

– stating that every professor teaches somebody. Now consider the following simple SPARQL query: SELECT $?x$ WHERE $(?x, \mathsf{teaches}, ?y)$.[1] According to the SPARQL entailment regimes standard [6], this query yields as result the empty set. □

This result is rather unintuitive: by the inclusion we know for certain that b teaches somebody. However, the SPARQL entailment standard requires that all values assigned to any variable must come from the RDF graph – thus treating distinguished variables (which are ultimately output) and non-distinguished variables (which are eventually projected out) in the same way. In contrast, the certain answer semantics retrieves all mappings on the distinguished variables that allow to satisfy the query in every possible model of the database and the ontology (yielding the certain answer $\mu = \{?x \to b\}$ in the above example).

The **goal of this work** is to introduce an intuitive certain answer semantics also for SPARQL under OWL 2 QL entailment with similarly favorable results as for CQ answering under DL-Lite$_\mathcal{R}$ (which provides the theoretical underpinning of the OWL 2 QL entailment regime).

The reason why for this purpose we cannot simply take over all the results from CQ answering under DL-Lite is that SPARQL provides some crucial extensions over

[1] Following [10], we use a more algebraic style notation, denoting triples in parentheses with comma-separated components, rather than the blank-separated turtle notation.

CQs. One of them is the OPTIONAL operator (henceforth referred to as OPT operator, for short). It allows the user to retrieve *partial solutions* in cases where no match for the complete query can be found, instead of failing to provide any solution. Observe that these queries are no longer monotone. Thus, the usual certain answer semantics (i.e., something is a certain answer if it is present in every model) turns out to be unsatisfactory:

Example 2. Consider the SPARQL query: SELECT $?x, ?z$ WHERE $(?x, \mathsf{teaches}, ?y)$ OPT $(?y, \mathsf{knows}, ?z)$ over the graph $G = \{(b, \mathsf{teaches}, c)\}$ and empty ontology $\mathcal{O}$. The query yields as only solution the mapping $\mu = \{?x \to b\}$. Clearly, also the extended graph $G' = G \cup \{(c, \mathsf{knows}, d)\}$ is a model of $(G, \mathcal{O})$. But in G', μ is no longer a solution since μ can be extended to $\mu' = \{?x \to b, ?z \to d\}$. Hence, there exists no mapping which is a solution in every possible model of $(G, \mathcal{O})$. □

In this work, we investigate further problems with a literal adoption of a certain answer semantics in the presence of the OPT operator, and propose a suitable modified definition for the class of *well-designed* SPARQL queries [10]. This modified semantics also requires an adaptation and extension of the known query answering algorithms for DL-Lite. We present two such modified algorithms for query evaluation.

Related Work to our findings includes the work our approaches are based upon [2, 4, 5, 6]. There is a huge body of results on CQ answering in DLs (cf. [4, 5, 9]). For SPARQL recent work [7] presents a stronger semantics.In [1], the authors describe a rewriting of SPARQL query answering under OWL 2 QL into Datalog$^{\pm}$. A slight modification allows them to remove the active domain semantics of variables, however this only applies to variables occuring in a single BGP. Libkin [8] also criticizes the standard notion of certain answers in case of non-monotone queries. Similar to his suggestion to use the greatest lower bounds in terms of informativeness, our approach chooses the most informative solutions as certain answers.

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Formalising the SECD machine with Nominal Isabelle

Gergely Buday

1 Introduction

This paper describes work in progress. My doctoral research goal is to prove the correctness of functional programs. I aim this as functional languages have formally defined operational semantics and this makes the problem more tractable than for imperative languages [21]. Especially this definedness makes advanced programming concepts easier to tackle.

CharguÃľraud [6] lists three ways of verifying functional programs: first is to define Hoare triples [9] , second is to define programs directly in a theorem prover, that is shallow embedding and the third is to define the semantics of the language in the logic of a theorem prover, use this definition to write programs and then prove correctness of these, this is called deep embedding.

The SECD machine consists of four parts, the Stack, the Environment, the Control stack and the Dump, hence the name. The stack consists of λ-expressions, used in expression evaluation. The environment is an associative array consisting of name-closure pairs where a closure is λ-term and an environment that stores all information needed to evalute that term. The control stack's elements are the remaining parts of the expression being evaluated. The dump is either empty or contains a backup of (S, E, C, D) triples, where D is the previous dump and this is saved when a closure is applied from the stack. It is restored when the control stack is empty but the dump is not.

My future plan is to prove the correctness of functional programs with respect to an SECD-style operational semantics. For this I need a formalisation of the SECD machine which was defined by Landin in his seminal paper [10] to evaluate λ-expressions. This paper describes a preliminary attempt to formalise and verify the SECD machine by contemporary tools. For a detailed introduction to the SECD machine see [14] [7].

A formalisation states that a classical SECD machine started with a λ-expression halts with a value that is the β-normal form of the original λ-expression.

2 Related work

I have found two formalisations in the literature: Ramsdell [19] and Graham [8]. Ramsdell's verification script is available for downloading [18] as is the NQTHM-1992 theorem prover [4] [3] that it uses. The theorem prover and the verification script on top of it ran without problems on current LISP implementations. This is of value concerning the age of the theorem prover and of the verification script.

For modelling the α-equivalence classes of the λ-calculus, Ramsdell used De Bruijn indices [5], which is a nameless representation of λ-expressions. That complicates the formalisation and the handling of the numeric indices permeates other parts of it so it would not be easy to replay the verification with other tools. The typeless nature of LISP also adds difficulties to the formalisation.

Graham had and fulfilled a much more ambitious goal: to verify a hardware implementation of the SECD machine, down to the register level. His verification is also available [11]. Because this formalisation is biased towards hardware implementation its SECD model is already complex, not to mention the way down to bitly details. So it again cannot be used directly.

Landin's original work is quite terse on the SECD machine itself and communicates more the idea than the details. Plotkin's classical paper [16] has a rigorous proof that I have chosen that to formalise. This is followed also by Field and Harrison [7],

3 Modeling the λ-calculus

To model the λ-calculus one has to equate terms that are isomorphic via a systematic renaming of variables: clearly $\lambda x.x$ denotes the same function as $\lambda y.y$. De Bruijn [5] invented a nameless, numeric binder mechanism. This unfortunately involves complex calculations on the indices and is counterintuitive regarding the way λ-expressions are understood usually. The standard way of proofs was to use abstract syntax and an informal argument on α-equivalence classes. There are a number of formal approaches though: [12] [17] [1] . I have chosen the theory of nominal sets [15] and its implementation, Nominal Isabelle [20] since it is a mature realisation and was applied to serious examples like the π-calculus [2].

Nominal sets are built on the theory of permutation groups, one of its main concepts is the *equivariant* function. Given a set X and a group G, an *action* of G on X is a function $G \times X \to X$ written $g \cdot x$ that satisfies $g \cdot (g' \cdot x) = (gg') \cdot x$ and $e \cdot x = x$. The full theory is described in [15].

Nominal Isabelle extends the Isabelle theorem prover [13] and the theory of nominal sets. It is a collection of Isabelle theory files that define the notions necessary for the theory of nominal sets, equivariant functions etc. .

It provides definitional mechanisms for nominal constructions that admit α-equivalence. Most of these lemmata can be verified by automatic methods.

Nominal Isabelle provides a theory of untyped λ-expressions which is based on the following definitions:

atom-decl *name*

nominal-datatype *lam* =
  Var name
| *App lam lam*
| *Lam x*::*name l*::*lam* **binds** *x* **in** *l* (*Lam* [-]. - [*100*, *100*] *100*)

so one can verify trivial α-equivalences easily:

lemma *Lam* [*x*]. *Var x* = *Lam* [*y*]. *Var y*
apply *auto*
done

4 Formalisation: a start

Plotkin [16] starts the definition of the SECD machine by a mutual definition of closures and environments. An environment maps variable names to closures, while a closure is a λ-term paired with an environment that contains all information to interpret the free variables of the term. Formally:

(1) If $x_1, \ldots, x_n$ are distinct variables and $Cl_i(i = 1, n)$ are closures, $\{\langle x_i, Cl_i\rangle | i = 1, n\}$ is in environments ($n \geqslant 0$).

(2) If E is an environment and M is a term such that $FV(M) \subseteq Dom(E)$, then $\langle M, E\rangle$ is a closure.

This can be translated to a nominal function definition. The current nominal package does not support type operators such as list, so I had to encode environment lists with the EmptyEnv ConsEnv constructor pair that imitates list definitions by Nil and Cons, where Nil is the empty list and Cons takes an element and a list and returns list topped with that element. The below verification is needed to ensure that env_lookup is an equivariant function.

nominal-datatype
  environment = *EmptyEnv*
      | *ConsEnv name closure environment*
  and *closure* = *Clos lam environment*

nominal-function
  env-lookup :: *environment* ⇒ *name* ⇒ *closure*
where
  env-lookup EmptyEnv x = *Clos* (*Var x*) *EmptyEnv*
| *env-lookup* (*ConsEnv v clos rest*) *x* =
    (*if* (*v* = *x*) *then clos else env-lookup rest x*)
  apply (*auto*)
  apply (*simp add*: *env-lookup-graph-aux-def eqvt-def*)

apply (*rule-tac y=a* **in** *environment-closure.exhaust*(*1*))
apply (*auto*)
done

The theorem env_lookup_graph_aux_def is the definition of the env_lookup function being defined, eqvt_def is the definition of equivariance: $eqvt\ ?f \equiv \forall p.\ p \cdot ?f = ?f$, while environment_closure.exhaust(1) is an induction theorem listing the two clauses of the environment datatype definition.

My preliminary experience is that Plotkin's proof is promising to use as a base for the formalisation and Nominal Isabelle is an adequate tool for such a formalisation. I aim to complete this formalisation.

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Dichotomy results for constraint satisfaction problems

Michael Kompatscher

A *constraint satisfaction problem (CSP)* is a computational problem where we are given a set of variables and a set of constraints on those variables. The task is to decide whether there is an assignment of values to the variables that satisfies all constraints. Computational problems of this type appear in many areas of computer science, for example in artificial intelligence, computer algebra, scheduling, computational linguistics, and computational biology.

Many CSPs can be modelled formally as follows: Let Γ be a structure with a (possibly infinite) domain D and a finite relational signature. Then the constraint satisfaction problem with *template* Γ, denoted by CSP(Γ), is the problem of deciding whether a given primitive positive sentence is true in Γ.

An important question is whether for finite templates every CSP is either in the P or NP-complete complexity class. If such a dichotomy theorem is true, then CSPs would provide one of the largest known subsets of NP which avoids NP-intermediate problems (under the assumption that P $\neq$ NP).

In [1] Schaefer proved that the dichotomy result is true for templates with Boolean domain. Special cases of his result include the NP-completeness of SAT and its two popular variants 1-in-3 SAT and Not-All-Equal 3SAT.

In his proof Schaefer used *primitive positive definable (pp-definable)* relations. A relation R is called primitive positive definable from a template Γ if $R(v_1, ..., v_k) \Leftrightarrow \exists x_1, ..., x_m C$ where C is a conjunction of relations from Γ over variables $\{x_1, ..., x_m, v_1, ..., v_k\}$.

A template Γ_1 is primitive positive definable from a template Γ_2, if all relations in Γ_1 are pp-definable from the relations in Γ_2. In this case CSP(Γ_1) reduces to CSP(Γ_2). Using this reduction Schaefer was able to show that every CSP with a Boolean domain that lies in P reduces to one of six known problems. All other problems are NP-hard, because SAT reduces to them.

A modern approach uses universal algebra and *polymorphisms* (cf. [2]). A polymorphism is a mapping $f : D^n \to D$ that preserves all the relations R in Γ meaning that

$$(x_{11}, ..., x_{1k}) \in R, ..., (x_{n1}, ..., x_{nk}) \in R \Rightarrow (f(x_{11}, ..., x_{n1}), ..., f(x_{1k}, ..., x_{nk})) \in R.$$

Given a template Γ, there is a surprisingly close connection between its polymorphisms and the computational complexity of CSP(Γ). The *polymorphism clone* $Pol(\Gamma)$

denotes the set of polymorphisms of Γ. Conversely, if O is a set of operations, then $Inv(O)$ denotes the set of relations having all operations in O as a polymorphism. Pol and Inv together build a Galois connection. For any template with a finite domain the set $Inv(Pol(\Gamma))$ is exactly the set of all relations that are pp-definable in Γ. So instead of studying primitive positive reducts we can study the lattice of polymorphism clones.

The universal algebraic approach even works, if we look at countably infinite, *ω-categorical* templates and introduce the topology of pointwise convergence on the clones. Then the Galois-closed objects are exactly the *closed* polymorphism clones. This result has already lead to a classification of the complexity of temporal complexity problems ([3]) and the Graph-SAT problems ([4]).

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MSO model checking of graphs with unbounded rank-width[*]

Eduard Eiben

Monadic second order logic (MSO) over graphs extends first order logic by variables that may range over sets of vertices (sometimes referred to as MSO_1 logic). For an MSO formula φ we can say that the MSO *model-checking problem* is the problem of deciding whether a graph G satisfies a formula φ. Many NP-hard graph problems can be naturally expressed as MSO model checking problems. Courcelle, Makowsky, and Rotics in [1] showed how to solve MSO model checking efficiently on graphs with bounded clique-width when a clique-decomposition is provided in the input. Later, Oum and Seymour introduced the notion of rank-width [8], which improves upon clique-width by allowing the efficient computation of decompositions while retaining the positive algorithmic results obtained for clique-width. Rank-width and clique-width are so far the most general parameters that allow fixed-parameter tractable (FPT; for further details on the topic see [2]) algorithms for solving MSO model checking.

An incomparable approach for solving graph problems is to use *modulators*. A modulator of a graph G to a specified base class $\mathcal{C}$ is a set of vertices whose deletion puts G in $\mathcal{C}$. The cardinality of a modulator to various tractable graph classes has long been used as a form of structure which can be exploited to obtain efficient algorithms for a range of important problems, and various popular notions such as vertex cover and feedback vertex set form special cases of modulators (see for instance the work of Fellows et al. [3] or Fomin et al. [4]). In other fields of computer science, modulators are often called backdoors and there are efficient algorithms using small backdoors for, e.g., Satisfiability and Constraints Satisfaction [5].

In our work, we investigate what happens when a graph contains a large but "well-structured" (in the sense of having bounded rank-width) modulator to some tractable class. Can such modulators still be exploited to obtain efficient algorithms? And is it even possible to find such modulators efficiently?

We introduce a new family of parameters that capture (roughly speaking) the minimum rank-width of any modulator to a graph class $\mathcal{H}$ and for any fixed $\mathcal{H}$ we call our parameter simply the well-structure number or $wsn^{\mathcal{H}}$. We show that the well-structure number is not only a more general parameter than the cardinality of modulators, but is in fact more general than rank-width itself. Furthermore, we

[*]Joint work with Robert Ganian and Stefan Szeider.

provide an FPT algorithm for finding such well-structured modulators to any graph class which can be characterized by a finite set of obstructions. This is achieved by building on the nearly linear-time algorithm for computing split-decomposition [6] in combination with the algorithm for computing rank-width [7].

Aside from developing algorithms utilizing well-structured modulators for individual problems, we also show that well-structured modulators can be used for the efficient model checking of any monadic second order sentence, as long as certain necessary conditions are met. Finally we turn our attention to the generalization of monadic second order logic toward minimization and maximization problems. We show that MSO optimization problems are not fixed-parameter tractable when parameterized by our parameter. This is surprising on one hand, since the fixed-parameter tractability of MSO optimization using classical width parameters traditionally follows from the fixed-parameter tractability of MSO model checking; on the other hand, since our parameters are strictly more general than rank-width, one cannot expect that every problem which is FPT parameterized by rank-width would remain FPT when parameterized by the well-structure number.

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Some Cardinal Invariants of the Generalized Baire Spaces*

Diana Carolina Montoya Amaya

Cardinal invariants of the continuum are cardinals describing mostly the combinatorial or topological structure of the real line. They are usually defined in terms of ideals on the reals, or some very closely related structure such us $\wp(\omega)/fin$. Typically, they assume values between $\aleph_1$, the first uncountable cardinal, and $\mathfrak{c} = |2^\omega| = |\omega^\omega| = |\mathbb{R}|$, the cardinality of the continuum, so they are uninteresting under the continuum hypothesis $\mathfrak{c} = \aleph_1$. However, in other models of set theory, they may assume different values and they provide means for characterizing the structure of the real line in such models.

The interaction between cardinal invariants in the classical Baire Space ω^ω has been deeply studied and it has contributed to the development of forcing techniques. A new developing area of interest nowadays is to consider its natural generalizations to the Generalized Baire Space κ^κ when κ is an uncountable cardinal. We will present a summary of the existent result on this matter as well as interesting open questions.

For example, the well known Roitman's Problem asks whether from $\mathfrak{d} = \aleph_1$ it is possible to prove that $\mathfrak{a} = \aleph_1$. Now, it is well known that $\mathfrak{d}$ and $\mathfrak{a}$ are independent, i.e. it is possible to prove the consistency of both $\mathfrak{a} < \mathfrak{d}$ and $\mathfrak{d} < \mathfrak{a}$. The problem that is still open is the consistency of $\mathfrak{d} = \aleph_1 < \mathfrak{a}$. In the uncountable case the analog of Roitman's Problem has already been resolved in the positive.

Theorem 1 (Theorem 2.1 in [3])**.** If κ is uncountable, regular cardinal and $\mathfrak{d}(\kappa) = \kappa^+$ then $\mathfrak{a}(\kappa) = \kappa^+$

Another example shows that, whereas in the countable case all these cardinal invariants are at least $\aleph_1$, in the uncountable case the splitting number at κ may not be at least κ^+.

Theorem 2 (Lemma 1 and 2 in [9])**.** $\mathfrak{s}(\kappa) \geq \kappa^+$ if and only if κ is weakly compact.

Finally we will show our current work in progress in the study of the cardinal invariants in the generalization of the well known Cichoń's Diagram. We consider just the cardinal invariants associated to the meager ideal on κ as well as the bounding and dominating numbers. We call this generalization Icho Diagram (See

*Joint work with Sy-David Friedman.

Fig.1). Moreover, we generalize classical models of set theory obtained as forcing extensions (κ-Cohen, κ-Eventually Different, κ-Hechler, etc.), and study the values that the cardinal invariants in the diagram take on them.

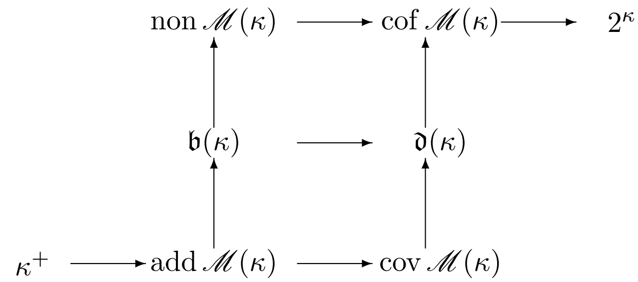

Figure 1: Icho diagram (for κ strongly inaccesible)

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Gaps In ω^ω

Marlene Koelbing

We work on the Baire space ω^ω, i.e. functions from $\mathbb{N}$ to $\mathbb{N}$. At the Baire space we use the order $\leq^*$ where $f \leq^* g$ iff $f(n) \leq g(n)$ for all but finitely many $n \in \mathbb{N}$. A (λ, μ)-pregap consists of two sequences $\langle f_\alpha | \alpha < \lambda \rangle$ and $\langle g_\beta | \beta < \mu \rangle$ such that the f_α-sequence is increasing, the g_β-sequence is decreasing and all the g_β are above all the f_α.

A pregap is a gap, if it can't be filled. That means, there exists no $h \in \omega^\omega$ such that $f_\alpha < h$ for each $\alpha < \lambda$ and $h < g_\beta$ for each $\beta < \mu$. The first question is, for which cardinals λ and μ (λ, μ)-gaps exist. Hadamard showed, that (ω, ω)-gaps do not exist. That can be proved by a construction which gives an h which fills the pregap. Hausdorff proved that an (ω_1, ω_1)-gap does exist. The construction of such a gap works by recursion. Hausdorff and Rothberger showed the connection to the bounding number $\mathfrak{b}$, which is $\mathfrak{b} = min\{\kappa : \text{ an } (\omega, \kappa) - \text{ gap exists}\}$.
Under MartinâĂŹs Axiom the cardinality of the continuum $\mathfrak{c}$ is also connected to the existence of gaps. Kunen showed, that under MartinâĂŹs Axiom, if there exists a (λ, μ)-gap for $\omega_1 < \lambda \leq \mu$ it follows, that $\lambda = \mathfrak{c}$ and additionaly, if MartinâĂŹs Axiom holds and $\lambda \neq \omega_1$ is a regular cardinal less than $\mathfrak{c}$, then there exists a $(\lambda, \mathfrak{c})$-gap.

The existence of $(\omega_1, \mathfrak{c})$-gaps and $(\mathfrak{c}, \mathfrak{c})$-gaps is independent from ZFC+MartinâĂŹs Axiom. Kunen showed, that it is possible, that both kinds of gaps exist and that it is also consistent, that neither $(\omega_1, \mathfrak{c})$-, nor $(\mathfrak{c}, \mathfrak{c})$-gaps exist. Yorioka proved, that it is consistent with ZFC+MartinâĂŹs Axiom, that there exist $(\mathfrak{c}, \mathfrak{c})$-gaps and no $(\omega_1, \mathfrak{c})$-gap or $(\omega_1, \mathfrak{c})$-gaps and no $(\mathfrak{c}, \mathfrak{c})$-gaps.
An other interesting question is, what happens to gaps under forcings, which do not collapse κ and λ to ω. Do they remain gaps or can they be filled? For each (ω_1, ω_1)-gap exists a Laver-forcing, which fills the gap and does not collapse ω_1 whenever a forcing-extension exists, in which the gap can be filled. Gaps which can be filled are called destructible. Moreover it can be proved, that the gap is destructible, iff the correlated forcing-order has the countable chain condition. Thus the question whether a gap is destructible can be reduced to a combinatorial question for a partial order.

The (ω_1, ω_1)-gap, constructed by Hausdorff is not destructible. But it is consistent, that a destructible (ω_1, ω_1)-gap exist. To show this, we can use a Cohen-forcing or the $\diamondsuit$-principle.

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The lifting problem for measure and category*

Anda Ramona Tănasie

In my PhD thesis I will investigate the existence of lifting homomorphisms of the Borel algebra modulo the ideal of meager sets on the generalized Baire space $\omega_1^{\omega_1}$. The existence of liftings in the case of the classical Baire space (ω^ω) was already investigated and it turned out to be independent of ZFC.

To obtain a model without liftings, Shelah defined *oracle-cc iterations* of forcing notions. The purpose of my thesis is to find an appropriate generalization of this notion and the corresponding iteration result.

Looking at the algebra of Borel sets modulo some ideal $\mathcal{I}$, we have a natural projection $p : \mathcal{B} \to \mathcal{B}/\mathcal{I}$, namely, the function mapping each Borel set to its equivalence class. We are interested whether a homomorphism $h : \mathcal{B}/\mathcal{I} \to \mathcal{B}$ such that $(h(x/_\mathcal{I}))_{/\mathcal{I}} = x/\mathcal{I}$ exists, equivalently, a homomorphism $h : \mathcal{B} \to \mathcal{B}$ with Kernel $\mathcal{I}$ such that $h(x) = x \bmod \mathcal{I}$. Such a homomorphism is called *lifting*.

When studying the ω-case, the "reals" will be $\mathbb{R}, [0,1], 2^\omega$ or ω^ω, whichever version is more convenient to work with in the current proof. The existence of a lifting has been studied for the ideal $\mathcal{I}_{mz}$ of Lebesgue measure zero sets and for the ideal $\mathcal{I}_{fc}$ of meager sets.

Under CH, there is a lifting for $\mathcal{B}/\mathcal{I}$ for both of these ideals. The proof essentially dates back to von Neumann: for each of these ideals, given a *density function* $d : \mathcal{B} \to \mathcal{B}$, there is a Borel lifting $h : \mathcal{B} \to \mathcal{B}$ such that $d(E) \subseteq h(E)$ for all $E \in \mathcal{B}$ ([1, Theorem 1.1]). For the existence of a density function, see [2].

A lifting homomorphism can also exist in a model of $\neg$CH. The construction of a model of ZFC $+ \neg$CH$+$ "there is a lifting for $\mathcal{B}/\mathcal{I}$" is due to Carlson: If $\aleph_2$ many Cohen reals are added to a model of CH, then in the extension $\mathfrak{c} = \aleph_2$ and there is a Borel lifting for $\mathcal{B}/\mathcal{I}$ ([1, Theorem 2.2]). The idea of his proof works for both $\mathcal{I}_{mz}$ and $\mathcal{I}_{fc}$.

Up to this point, nothing is known about the existence of lifting homomorphisms with $\mathfrak{c} > \aleph_2$.

The most impressive result concerning this problem is due to Saharon Shelah ([3], [5] and [4]) and concludes that the existence of such a homomorphism is independent of ZFC. He showed the consistency of ZFC $+\ \mathfrak{c} = \aleph_2+$ "there is no lifting for $\mathcal{B}/\mathcal{I}_{mz}$". He presented the proof for the ideal of Lebesgue measure zero sets, but the same proof (as Shelah himself pointed out) works for the ideal of meager sets as well.

Using an ω_2 stage finite support iteration of ccc forcing notions of size $\aleph_1$, Shelah obtained a model $V^{P_{\omega_2}}$, where there is no lifting. Obviously, in this model, CH has to fail.

*Joint work with Sy-David Friedman.

Shelah argued that a poset killing the lifting exists, that is, a poset that adds a Borel set $X \subseteq \omega^\omega$ such that $h(X)$ cannot be defined. The oracle-cc theory comes to use at this point for ensuring no possible value for $h(X)$ is added in the iteration, thus there will be no way to extend the function we had at an initial stage α to a lifting in the final model.

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Unified Correspondence as a Proof-Theoretic Tool*

Apostolos Tzimoulis

This presentation focuses on the formal connections between correspondence phenomena, well known from the area of modal logic, and the theory of display calculi, originated by Belnap [1].

Sahlqvist correspondence theory. Sahlqvist theory [14, 15] is among the most celebrated and useful results of the classical theory of modal logic, and one of the hallmarks of its success. It provides an algorithmic, syntactic identification of a class of modal formulas whose associated normal modal logics are *strongly complete* with respect to *elementary* (i.e. first-order definable) classes of frames.

Unified correspondence theory. Correspondence theory is currently a very active field of research. This field has significantly broadened its scope in recent years, extending the benefits it originally imparted to modal logic to a plethora of logics. These logics include, among others, intuitionistic and lattice-based (modal) logics [7], substructural logics [8], non-normal modal logics [11, 13], hybrid logics [3], mu-calculus [5, 4].

The breadth of this work has stimulated many and varied applications. Some are closely related to the core concerns of the theory itself, such as the understanding of the relationship between different methodologies for obtaining canonicity results [12], or of the phenomenon of pseudocorrespondence [9]. Other, possibly surprising applications include the dual characterizations of classes of finite lattices [11]. These and other results have given rise to a theory called *unified correspondence* [6].

Tools of unified correspondence theory. The most important technical tools in unified correspondence are: (a) a very general syntactic definition of the class of Sahlqvist formulas, which applies uniformly to each logical signature and is given purely in terms of the order-theoretic properties of the algebraic interpretations of the logical connectives; (b) the algorithm ALBA, which effectively computes first-order correspondents of input term-inequalities, and is guaranteed to succeed on a wide class of inequalities (the so-called *inductive* inequalities) which, like the Sahlqvist class, can be defined uniformly in each mentioned signature, and which properly and significantly extends the Sahlqvist class.

*Joint work with Giuseppe Greco, Minghui Ma, Alessandra Palmigiano and Zhiguang Zhao.

Unified correspondence and display calculi. The proposed talk focuses on an entirely different type of application of unified correspondence: the identification of the syntactic shape of axioms which can be translated into analytic rules of a display calculus. A rule is called *analytic* if adding it to a display calculus preserves Belnap's cut-elimination theorem. The connections between Sahlqvist theory and display calculi have been seminally observed by Marcus Kracht in [10], in the context of his characterisation of those formulas of the language of basic modal logic (which he calls *primitive formulas*) which can be effectively transformed into structural rules of display calculi.

Contributions. The two tools of unified correspondence can be put to use to generalise Kracht's transformation procedure from axioms into analytic rules. This generalisation concerns more than one aspect. Firstly, in the same way in which the definitions of Sahlqvist and inductive inequalities can be given uniformly in each logical signature, the definition of primitive formulas/inequalities is introduced for any logical framework the algebraic semantics of which is based on distributive lattices with operators. Secondly, in the context of each such logical framework, we introduce a hierarchy of subclasses of inductive inequalities, progressively extending the primitive inequalities, the largest of which is the class of so-called *analytic inductive inequalities*. This class significantly generalises the class of primitive formulas/inequalities. We provide an effective procedure, based on ALBA, which transforms each analytic inductive inequality into an equivalent set of analytic rules. Moreover, we show that any analytic rule can be effectively and equivalently transformed into some analytic inductive inequality.

Structure of the talk. We give an informal presentation of how the algorithm ALBA computes the first-order correspondent of input inequalities. In this discussion we highlight that the soundness of ALBA-computations is based on the order theoretic properties of the interpretation of the logical connectives. This motivates the identification of the syntactic shape (the so-called *inductive inequalities*), in any logical signature based on distributed lattices, on which ALBA is guaranteed to be successful. In any such signature, the theory of display calculi provides an associated display calculus which we briefly introduce. We discuss the subclass of inductive inequalities, the so-called *analytic inductive inequalities*, and we show by way of examples how ALBA transforms these into analytic rules of the display calculus previously introduced. Time permitting, we illustrate a different but equivalent procedure to generate analytic rules, described in [2]. Interestingly, notwithstanding the fact that this procedure is motivated in purely proof theoretic terms, there are clear similarities between these two procedures. A systematic justification of these similarities is a current research direction.

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Logical Metatheorems for Abstract Spaces axiomatized in Positive Bounded Logic

Daniel Günzel

We extend the range of the logical metatheorems due to Kohlenbach to cover $C(K)$, abstract L^p-spaces, Banach lattices, bands of $L^p(L^q)$-Bochner spaces and to all normed structures axiomatized in positive bounded logic due to Henson-Iovino. We also present a proof-theoretic analogon of the model-theoretic use of ultrapowers of Banach spaces: a generalized uniform boundedness principle. This correspondence is exemplified by several theorems having a model-theoretic and a proof-theoretic counterpart.

1 Logical Metatheorems for Abstract Spaces axiomatized in Positive Bounded Logic

In this work my supervisor Prof. Kohlenbach and I extend the range of the logical metatheorems developed by [5] which have found 50 applications in nonlinear analysis, ergodic theory, partial differential equations and optimization published in the last two decades. Our extension includes $C(K)$, abstract L^p-spaces, Banach lattices, bands of $L^p(L^q)$-Bochner spaces and all normed structures axiomatizable in positive bounded logic.

While a "regular" theorem has mathematical objects as input such as functions, fixed points etc., a metatheorem takes a proof of a theorem as input. More specifically, a proof, formalized in a suitable logical framework, is used to obtain new constructive results such as explicit quantitative bounds. Our formal framework for abstract normed spaces X is called $\mathcal{A}^\omega[X, \|\cdot\|]$, which is an extension of Peano Arithmetic to higher types (with base types $\mathbb{N}$ and X). As axioms we have the axiom of dependent choice for all types, constants and universal axioms of normed spaces and we represent real numbers by functions $f : \mathbb{N} \to \mathbb{N}$. The extensionality axiom is not included in the theory, we only have the quantifier-free rule of extensionality available. One of the key concepts in the proof of the metatheorem is majorizability, going back to Howard (73), which is defined for the base type $\mathbb{N}$ and the abstract type X as follows: $n \gtrsim_{\mathbb{N}} m := n \geq m$ and $x^* \gtrsim_X x := x^* \geq \|x\|$ with $x^* \in \mathbb{N}$. For higher types this is extended in a hereditary way. We read $x^* \gtrsim_\rho x$ as "x^* majorizes x". In practice majorants are often easy to compute, for example let $f : X \to X$ be a nonexpansive function, *i.e.* $\forall x, y \in X \, (\|f(x) - f(y)\| \leq \|x - y\|)$. Then f is majorized by: $f^* : \mathbb{N} \to \mathbb{N}$ with $f^*(n) := n + \lceil \|f(0)\| \rceil$.

We discuss positive bounded logic (having the same expressive power as continuous logic due to Keisler and adapted by [1]), which is a fragment of first-order logic due to [3, Henson-Iovino], and is used in model theory to axiomatize classes of normed structures which are closed under forming ultrapowers and ultraroots. Continous first order logic is the main tool used by the above authors (see [2]), but since the expressive power is the same we have chosen to consider the syntax of positive bounded logic. It is a many-sorted logic, with a special sort for $\mathbb{R}$. All functions are assumed to be uniformly continuous (which is not necessary in our framework) and there are only bounded quantifiers: $\exists_r xA := \exists x(\|x\| \le r \wedge A)$ and $\forall_r xA := \forall x(\|x\| \le r \rightarrow A)$ (with $r \in \mathbb{Q}$ positive). The prime formulas are of type $t \le r$ and the connectives are $\vee, \wedge$. A very important concept is the approximate satisfaction of a set of positive bounded formulas. If φ is a positive bounded formula, $\forall k \in \mathbb{N}\varphi(k)$ expresses all 2^{-k} approximations of φ. We show that not only each positive bounded formula, which we define as a syntactic class $\mathcal{PBL}$ in our framework, is admissible as an axiom in the metatheorem, but also $\forall k \in \mathbb{N}\varphi(k)$.

Using a uniform boundedness principle (which fails in the full set-theoretic model) of the following type (in fact we use a more technically involved axiom):

$$\Sigma^0_1\text{-UB}^X : \forall y^\rho (\forall x \le_\rho y \exists z \in \mathbb{N} A_\exists(x,y,z) \rightarrow \exists z^* \forall x \le_\rho y \exists z \le_{\mathbb{N}} z^* A_\exists(x,y,z)),$$

where $A_\exists$ is an $\exists$-formula and the type ρ may be X or $\mathbb{N} \rightarrow \mathbb{N}$, we show that a formula φ of the class $\mathcal{PBL}$ is equivalent to $\forall k \in \mathbb{N}\varphi(k)$. Now we come to one of our main theorems, which we formulate for improved readability only for a special case:

Theorem (Logical Metatheorem)**.** Let Θ be a set of sentences of the class $\mathcal{PBL}$ and let $\Theta_\approx$ be the set of all approximations of Θ. Let $A_\exists(f,x,T,v)$ be an $\exists$-formula containing only f, x, T, v as free variables. Assume

$$\mathcal{A}^\omega[X, \|\cdot\|] + \Theta + \Sigma^0_1\text{-UB}^X \vdash \forall f^{\mathbb{N}\rightarrow\mathbb{N}}, x^X, T^{X\rightarrow X} \exists v^{\mathbb{N}} A_\exists(f,x,T,v)$$

then one can extract a total (bar recursive) computable functional $\Phi : \mathbb{N}^{\mathbb{N}} \times \mathbb{N} \times \mathbb{N}^{\mathbb{N}} \rightarrow \mathbb{N}$ such that the following holds in all nontrivial normed spaces X satisfying the axioms $\Theta_\approx$: for all $f \in \mathbb{N}^{\mathbb{N}}$, $x \in X$, $T \in X^X$ and $f^* \in \mathbb{N}^{\mathbb{N}}$, $x^* \in \mathbb{N}$, $T^* \in \mathbb{N}^{\mathbb{N}}$ if $f^*, x^*, T^* \gtrsim f, x, T$ then

$$\exists v \le \Phi(f^*, x^*, T^*) A_\exists(f,x,T,v).$$

In [5] one finds similar metatheorems for the following spaces: (complete) metric spaces, CAT(0)-spaces, W-hyperbolic spaces, uniformly convex spaces, Hilbert spaces. We extend this list to cover abstract L^p-spaces ($1 \le p < \infty$), Banach lattices, C(K)-spaces of continuous real valued functions on an abstract compact topological space K and bands of $L^p(L^q)$-Bochner spaces. This is done in the language of Banach lattices using characterizations of Kakutani, Bohnenblust, Nakano, Gordon, Krivine, and more recently for $L^p(L^q)$ [4, Henson-Raynaud]. For instance, by adding the inequality

$$\|x \sqcup y\|^p \leq \|x\|^p + \|y\|^p \leq \|x+y\|^p\,, \qquad \text{for all positive } x, y \in X \tag{1}$$

where $x \sqcup y$ denotes the supremum of x and y (elements of a Banach lattice X), it is known that any model of this theory (axiomatization of a Banach lattice plus (1)) is isometrically lattice isomorphic to an L^p-space. The same technique, only changing the norm inequality, is used to axiomatize $C(K)$ in terms of Banach lattices. Henson and Raynaud presented in [4] an infinite list of axioms, also using Banach lattices, which axiomatizes bands of $L^p(L^q)$-Bochner spaces. In our formal framework we can express their list of axioms by one sentence.

Finally, we provide a list of applications of the uniform boundedness principle of the following kind: $\mathcal{A}^\omega[X, \|\cdot\|] + \Sigma^0_1\text{-UB}^X \vdash X$ is strictly convex $\rightarrow X$ is uniformly convex. Using ultrapowers the corresponding result is, that a Banach space X is uniformly convex if and only if its ultrapower is strictly convex. More examples of the correspondence of uniform boundedness to the model-theoretic use of ultrapowers are:

smoothness + Σ^0_1-UB^X	= uniformly smoothness
normed space + Σ^0_1-UB^X	= Banach space
p(n) convexity + Σ^0_1-UB^X	= P(n)-convexity

As a summary we have established an effective and quantitative form of results treated so far only in model theory using ultrapowers for the proof-theoretic framework. In [2] the authors have proven a completeness theorem for continuous logic, which shall not be confused with our applied proof-theoretic approach via the above metatheorem.

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Proof Mining in Nonlinear Analysis

Daniel Körnlein

Proof mining is a research program in proof theory that was inspired by Kreisel's central question from the 1950's:

"What more do we know if we have proved a theorem by restricted means than if we merely know that it is true?"

The additional content inherent in proofs in certain restricted theories can be of quantitative (e.g. convergence rates, rates of metastability, moduli of uniqueness) or of qualitative nature (weakened hypotheses, uniformity results). The endeavors over the past 20 years in this vein can be divided into two categories; on the one hand, general metatheorems *guarantee* the extractability of quantitative data under general requirements on the logical form of the theorem in question and the proof-theoretic principles used in its proof. On the other hand, there have been numerous applications of these metatheorems to concrete examples in a variety of fields, including fixed point theory, ergodic theory, continuous optimization, best L_1-approximation and abstract Cauchy problems. For a comprehensive treatment see Kohlenbach [1].

The central methods used in proof mining are (variants of) Gödel's functional interpretation (Dialectica), negative translations and majorization. Applying (a combination of) these methods to concrete proofs p of a theorem A results in a new proof of p' of a new theorem A', where A' exhibits the additional content. Moreover, A' will imply trivially the original statement.

The talk will focus on applications of this methodology to metric fixed point theory. In particular, we will examine from a proof-theoretic perspective a convergence result due to Yamada [6]:

Theorem 1. Let H be a Hilbert space, $T : H \to H$ be a nonexpansive mapping with nonempty set of fixed points $Fix(T)$. Suppose that for $\eta, \kappa > 0$, a mapping $F : H \to H$ is $\kappa-$Lipschitzian on $T(H)$ and satisfies the inequality $\langle Fx - Fy, x - y \rangle \geq \eta \|x - y\|^2$ for all $x, y \in T(H)$. Then, for any μ and p satisfying $0 < p < 1$ and $0 < \mu < 2\eta/\kappa^2$, the sequence (u_n) defined by $u_n := T(u_{n-1}) - \mu n^{-p} F(T(u_{n-1}))$ converges in norm to the unique solution of the Variational Inequality Problem *VIP*:

VIP: Find $u^* \in Fix(T)$ such that $\langle v - u^*, F(u^*) \rangle \geq 0$ for all $v \in Fix(T)$.

The Cauchyness of an iteration (x_n) with respect to a metric d is translated by the functional interpretation to

$$\forall k \in \mathbb{N} \forall g : \mathbb{N} \to \mathbb{N} \exists n \in \mathbb{N} \forall i, j \in \{n, n+1, \dots n + g(n)\} \left(d(x_i, x_j) < 2^{-k} \right),$$

which coincides with the Herbrand normal form (in this case!) and was recently rediscovered by Tao under the name of metastability [4, 3]. The additional, quantitative content then consists of a bound $\Phi(g,k)$ on the existential quantifier of the statement, called a *rate* of metastability.

Our analysis of this theorem will highlight the modularity of the functional interpretation in that it is sound with respect to the modus ponens. In fact, to obtain a rate of metastability, one can reuse an earlier analysis [2] of a theorem due to Wittmann [5] once one has extracted the appropriate moduli from the given proof that (an approximate version of) *VIP* has a solution.

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On the Why and How of implicit conflicts in Abstract Argumentation*

Christof Spanring

1 Background and Motivation

This work deals with Dung-style abstract argumentation as first introduced in [4]. In a nutshell we use some structure $\mathcal{F} = (\mathcal{X}, \mathcal{A})$ called argumentation framework (AF), consisting of a set of arguments $\mathcal{X}$ and an attack relation on these arguments $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{X}$. Here arguments might refer to natural language arguments such as $a =$"Homeopathy is Scam" or $b =$"NHS supports Homeopathy", whereby in this constellation naturally b attacks a but not the other way around. In graphical representations we identify nodes with arguments and directed edges with attacks.

Example 3. Consider the AF as graphically represented in Figure 1. Here we have that the only self-defending argument is d. If we accept d, then c is rejected, thus d defends b. Now if we accept b then a is rejected.

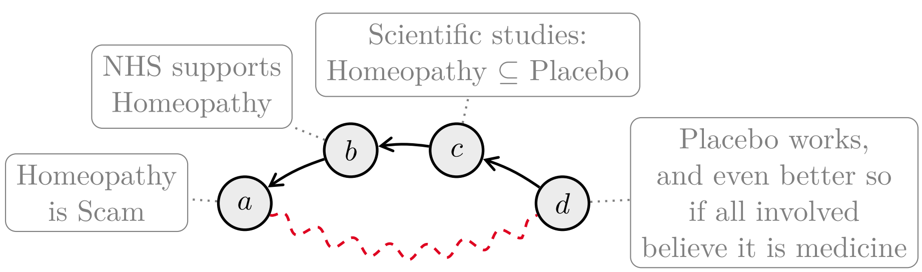

Figure 1: Implicit Conflicts in natural language arguments.

2 Argumentation Semantics

To elaborate on a meaning of truth we use argumentation semantics (see [1] for an established selection of such). Formally an argumentation semantics is a mapping σ, assigning sets of reasonable arguments to each AF, $\sigma(\mathcal{F}) \subseteq \mathcal{P}(\mathcal{X})$. Intuitively each semantics builds upon a selection of semantical properties, depending only on the

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attack relation and not on names or some underlying meaning of the arguments. The sets $S \in \sigma(\mathcal{F})$ are called σ-extensions of $\mathcal{F}$. For extensions we are interested in such properties as consistency (conflict- or attack-freeness), admissibility (self-defense), maximality, stability (assigning to each argument exactly one status from $\{accept, attack\}$), range-maximality (maximal area of influence), directionality (prefer arguments that occur earlier in argumentation paths) and more.

Example 4. For Example 3 now observe that despites a and d not attacking each other, they do not occur together in the only maximal admissible extension $\{b, d\}$. Also observe that simply adding an attack (d, a) in this case does not change the extensions but resolve that implicit conflict.

On the language level we can say that homeopathy might still be scam regardless of whether placebo works or not. This objection aside, on a purely abstract level, however, the question occurs whether implicit conflicts can always be expressed explicitly through modification of the attack relation without altering the given extension-sets.

3 Implicit Conflicts

Quite some effort has already been put into investigating relations between various argumentation semantics, see e.g. [2, 6, 5]. In this work we discuss and answer questions recently posed in [3], and further investigate meaning and existence conditions of implicit conflicts.

Definition 1. For a given AF $\mathcal{F} = (\mathcal{X}, \mathcal{A})$ and semantics σ we say that arguments x, y are in *conflict* if there is no extension $S \in \sigma(\mathcal{F})$ such that $x, y \in S$. The conflict is called *explicit* if $(x, y) \in \mathcal{A}$ or $(y, x) \in \mathcal{A}$, otherwise the conflict is called *implicit*.

It is easy to see that for some AFs some conflicts are implicit. For (maximal) conflict-free sets it is also easy to see that implicit conflicts occur only in relation with self-attacking arguments. In [3] an AF was presented showing that for maximal admissible and range-maximal admissible semantics and some AFs there is no semantically equivalent AF with only explicit conflicts. Hence some conflicts are implicit by their very nature and can not be explicitly represented via attacks.

Definition 2. Two AFs $\mathcal{F}$ and $\mathcal{G}$ are considered as semantically equivalent with respect to semantics σ if they share the same sets of arguments and $\sigma(\mathcal{F}) = \sigma(\mathcal{G})$.

In this work we are interested in the mechanisms allowing implicit conflicts and in comparisons of semantics with their way of incorporating and dealing with (implicit) conflicts. When comparing extension-sets it occurs that more sophisticated semantics (allowing a broader range of extension-sets) are more likely to provide implicit conflicts, we thus regard implicit conflicts as a feature of expressiveness, and compare various semantics with respect to this property.

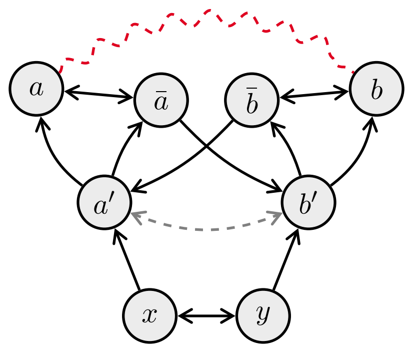

Figure 2: Example for implicit conflicts caused by directionality.

Example 5. Take the AF from Figure 2 and directionality and maximal conflict-freeness as semantical properties. As the set $\{x, y\}$ has no incoming attacks, wlog. we accept x. Then a' is rejected and $\{a, \bar{a}\}$ has no incoming attacks. If we accept a then $\bar{a}$ is attacked, b' has no more incoming attacks and can be accepted as well. But then $\bar{b}$ as well as b are rejected. Thus accepting a means that b can not be accepted and vice versa.

If we add an attack (b, a), then $\{a, \bar{a}\}$ receives one more incoming attack and as second component we thus need to consider $\{a, \bar{a}, b', \bar{b}, b\}$ instead, leading to the additional extension $\{x, a, \bar{b}\}$. Observe that this AF also has an implicit conflict between a' and b' which can be made explicit without altering the extension-sets. The conflict between a and b however appears to be immanently implicit.

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Extracting consequence relations from abstract argumentation frames

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In the theory on abstract argumentation introduced by Dung [1], arguments are seen as abstract entities that may attack each other. Dung defines an argumentation framework as a pair $(Ar, \to)$ where Ar is a set of arguments and $\to$ an attack relation over Ar; in particular if $a, b \in Ar$, $a \to b$ means that the argument a attacks the argument b. An argumentation framework thus is just a directed graph in which the arguments are the vertices and the attack relation defines the edges.

Instead of considering arguments as abstract entities, we depict them as propositional formulas over the classical language ($\supset, \wedge, \vee$ and $\bot$). Often (see, e.g., [3]) an argument is identified with an ordered pair $A = (\Phi, \phi)$ where Φ is a set of propositional formulas, called the support, and ϕ a formula such that $\Phi \models_{CL} \phi$, called the claim. We will consider only the special case in which the support and the claim coincide and these arguments may simply be identified with propositional formulas. We say that an argumentation frame $(Ar, \to)$ is *logically closed* if for any argument $a \in Ar$, all the subformulas of a are in Ar. Under this formalization, some conditions apply to the arguments and some of them follow from what we have called postulates. We obtain, for example, the following:

Postulate 1 (P_1): If $a \models_{CL} b$ and $x \to b$, then $x \to a$.

From P_1, since $A \wedge B \models_{CL} A$ and $A \wedge B \models_{CL} B$, whenever $x \to A$ (or $x \to B$), then $x \to A \wedge B$. Such conditions that follow from the general ones are called *weak*. Making further assumptions on the behaviour of the arguments, we define also other conditions denominated as *strong*. An example of a strong condition is the following: if $x \to a \wedge b$, then either $x \to a$ or $x \to b$. Similar conditions can be defined also for the other connectives and we say that $(Ar, \to)$ is *logically saturated* if its arguments satisfy these conditions.

Definition 3. Let $F = (Ar, \to)$ be a logically closed and saturated argumentation framework, $A \in Ar$ and $\Gamma \subseteq Ar$. We say that $\Gamma \models^F_{att} A$ iff whenever $x \to A$ for some $x \in Ar$, then there exists $\gamma \in \Gamma$ such that $x \to \gamma$; i.e. if x attacks the consequent (A), then x attacks at least one element of the premise (Γ). We say that $\Gamma \models_{att} A$ iff $\Gamma \models^F_{att} A$ for all frames F.

For the entailment relation just introduced the following theorem holds:

Theorem 1. $\Gamma \models_{att} A$ iff $\Gamma \models_{CL} A$, where $\models_{CL}$ stands for the consequence relation for classical logic.

In the proof of the Theorem we use that Gentzen's sequent calculus (see [2]) is sound and cut-free complete with reference to classical logic and that its logical rules are invertible.

Following Dung's definitions [1], we say that an argument $a \in Ar$ is *acceptable* with respect to a set $\Gamma \subseteq Ar$ of arguments iff for each argument $b \in Ar$, if $b \to a$ then exists $\gamma \in \Gamma$ such that $\gamma \to b$.

Furthermore, Dung defines the characteristic function F_F on an argumentation framework as a function $F_F : 2^{Ar} \to 2^{Ar}$ such that

$$F_F(\Gamma) = \{a | a \textit{ is acceptable with respect to } \Gamma\}.$$

Therefore we can define another consequence relation $\models_{adm}$ whose correspondent operator C^F_{adm} coincides with F_F, i.e. for every argumentation framework F logically closed and saturated, and $\Gamma \subseteq Ar$, $C^F_{adm}(\Gamma) = F_F(\Gamma)$.

Definition 4. Let $F = (Ar, \to)$ be a logically closed and saturated argumentation framework, $A \in Ar$ and $\Gamma \subseteq Ar$. We say that $\Gamma \models^F_{adm} A$ iff whenever $x \to a$, for some $x \in Ar$, then there exists $\gamma \in \Gamma$ such that $\gamma \to x$. We say that $\Gamma \models_{adm} A$ iff $\Gamma \models^F_{att} A$ for all frames F.

For the consequence relation just introduced the following theorem holds:

Theorem 2. $\Gamma \models_{adm} A$ iff $\Gamma \models_{CL} A$, where $\models_{CL}$ stands for the consequence relation for classical logic.

Other definitions of consequence relations similar to $\models_{att}$ and $\models_{adm}$ can be introduced and for each of them a similar theorems to the Theorems 2 and 4 can be proved.

These results are a starting point for a more general analysis of nonmonotonic reasoning. A very weak form of nonmonotonic consequence can be recovered from logical saturated argumentation frameworks as follows: $A_1, \ldots, A_n \models_{nm} B$ iff no A_i ($i \in \{1, \ldots n\}$) attacks B. This consequence relation can be strengthened by additionally requiring that for every argument x such that $x \to B$, $A_i \to x$ for some $i \in \{1, \ldots, n\}$.

In any case the overall project proposed here is to systematically explore which (monotonic and nonmonotonic) logic corresponds to which of the various alternative consequence relations extracted from argumenation frameworks.

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The Expressive Power of k-ary Inclusion-Exclusion Logic

Raine Rönnholm

The origin of *inclusion-exclusion logic* lies in the notions of dependence and imperfect information. One of the first approaches in this area was *IF-logic* (independence friendly logic) by Hintikka and Sandu [9]. The truth for IF-logic was originally defined by using semantic games of imperfect information ([10]), but an equivalent compositional semantics was presented later by Hodges [11]. However, in a compositional approach it is not sufficient to consider single assignments, but instead sets of assignments which are called *teams*. They can be seen as parallel positions in a semantic game, or can also be interpreted as information sets or as databases ([14]).

By using similar *team semantics*, Väänänen [14] introduced *dependence logic* in which the dependencies of the values of terms are interpreted on the level of atomic formulas with *dependence atoms*. A similar approach is taken in *independence logic* presented by Grädel and Väänänen [7]. These logics have been recently studied a lot with an attempt to formalize the dependency phenomena in different fields of science, such as database dependency theory ([13]), belief presentation ([3]) and quantum mechanics ([12]).

Inclusion and exclusion logics were first presented by Galliani [4] in 2012. They extend first order logic with *inclusion and exclusion atoms* similarly as dependence atoms in dependence logic. The semantics for these atoms correspond to inclusion and exclusion dependencies in database theory ([4]): Suppose that $\vec{t_1}, \vec{t_2}$ are k-tuples of terms and X is a team. The k-ary inclusion atom $\vec{t_1} \subseteq \vec{t_2}$ says that the values of $\vec{t_1}$ are included in the values of $\vec{t_2}$ in X. The k-ary exclusion atom $\vec{t_1} \mid \vec{t_2}$ analogously says that $\vec{t_1}$ and $\vec{t_2}$ get distinct values in X. Our main topic of interest is inclusion-exclusion logic in which we use both of these atoms that have a dualistic relationship.

Without arity bounds exclusion logic has been shown to be equivalent with dependence logic ([4]) which captures *existential second order logic*, ESO, on the level of sentences ([14]). Inclusion logic is not comparable with dependence logic in general ([4]), but captures *positive greatest fixed point logic* on the level of sentences, as shown by Galliani and Hella [6]. Hence exclusion logic captures NP, and inclusion logic captures PTIME over models with linear order. Galliani [4] has shown that inclusion-exclusion logic is equivalent with independence logic, and that exactly ESO-definable properties of teams can be defined with inclusion-exclusion logic.

It is natural to ask how does the arity of atoms effect the expressive power of these logics. Hannula [8] has shown that inclusion logic has a strict arity hierarchy over graphs, while Durand and Kontinen [2] have shown that, on the level of sentences, k-ary dependence logic captures the fragment of ESO in which at most $(k-1)$-ary functions can be quantified. Galliani, Hannula and Kontinen [5] have shown that the latter holds also for k-ary independence logic. This however, does not resolve the expressive power of k-ary inclusion-exclusion logic, INEX[k], since the translation from it to independence logic does not respect the arities of atoms.

Our first theorem is that every formula of EXC[k] can be expressed with a formula of k-ary ESO, ESO[k] (in which at most k-ary relation variables can be quantified). The idea of this compositional translation is that for each occurrence of an exclusion atom $\vec{t}_1 \mid \vec{t}_2$ we quantify a separate k-ary relation variable that gives limits to the values that the tuple $\vec{t}_1$ can get and $\vec{t}_2$ cannot. We present a similar translation for k-ary inclusion logic, INC[k], and combine these to get a translation from INEX[k] to ESO[k].

For the other direction we show that all ESO[k]-formulas with at most k-ary free relation variables can be expressed with a formula of INEX[k]. In this very natural translation the quantified k-ary relation variables P_i are just replaced with k-tuples $\vec{w}_i$ of quantified variables. Then we simply replace atomic formulas of the form $P_i\vec{t}$ with inclusion atoms $\vec{t} \subseteq \vec{w}_i$ and formulas of the form $\neg P_i\vec{t}$ dually with exclusion atoms $\vec{t} \mid \vec{w}_i$.

For this latter translation we also need a new operator called *term value preserving disjunction* which can be defined in INEX[k]. This is a useful operator for any logic with team semantics, since the splitting of the team when evaluating disjunctions tends to lose information about the values of terms. By being able to preserve values, we can use k-tuples of first order variables to simulate k-ary relation variables as we do in our translation from ESO[k] to INEX[k].

From these results it follows that, on the level of sentences, INEX[k] captures the expressive power of ESO[k]. In particular, by using only unary inclusion and exclusion atoms we get the expressive power of *existential monadic second order logic*, EMSO. We also get a strict arity hierarchy for inclusion-exclusion logic on the level of sentences, since the arity hierarchy for ESO is known to be strict as shown by Ajtai [1] in 1983.

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